1.Harvard Law School courses often have assigned seating to facilitate the “Socratic method.” Suppose that there are 100 first year Harvard Law students, and each takes two courses: Torts and Contracts. Both are held in the same lecture hall (which has 100 seats), and the seating is uniformly random and independent for the two courses?

(a) Find the probability that no one has the same seat for both courses (exactly; you should leave your answer as a sum).

(b) Find a simple but accurate approximation to the probability that no one has the same seat for both courses.

(c) Find a simple but accurate approximation to the probability that at least two students have the same seat for both courses.

Ans. (A) There are 100! possible seating arrangements for the two courses, and only one of them has no students with the same seat in both courses. Therefore, the probability that no one has the same seat for both courses is 1/100!.

(B) The probability that no one has the same seat for both courses is very small, so we can approximate it with the probability that no two students have the same seat in both courses. There are 100 choose 2 = 4950 possible pairs of students who could have the same seat in both courses. Each of these pairs has a probability of 1/100^2 = 1/10000 of having the same seat in both courses. Therefore, the approximate probability that no two students have the same seat in both courses is 1 - 4950/10000 = 9951/10000.

(C) The probability that at least two students have the same seat for both courses is 1 - 1/100! = 1 - 9.333e-301. This is a very small probability, so we can approximate it with the probability that at least one pair of students has the same seat in both courses. This probability is 1 - 9951/10000 = 49/10000 = 0.049.

2. There are 100 passengers lined up to board an airplane with 100 seats (with each seat assigned to one of the passengers). The first passenger in line crazily decides to sit in a randomly chosen seat (with all seats equally likely). Each subsequent passenger takes his or her assigned seat if available, and otherwise sits in a random available seat. What is the probability that the last passenger in line gets to sit in his or her assigned seat?

Ans. The probability that the last passenger gets to sit in their assigned seat is 1/2.

There are two possible outcomes:

1. The first passenger takes the last passenger's seat. In this case, the last passenger will not be able to sit in their assigned seat.
2. The first passenger takes a different seat. In this case, the last passenger will be able to sit in their assigned seat.

Since the first passenger has an equal chance of taking either seat, the probability that the last passenger gets to sit in their assigned seat is 1/2.